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Addendum

Addendum to "Groups of ribbon knots"

Ka Yi Ng*

145 Lincoln Avenue, Apt. 5W, Staten Island, NY 10306, USA

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Abstract

The inductive step that was described in the proof of Theorem 3.2 of Ng (Topology 37(2) (1998) 441) is clarified.

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The purpose of this document is to clarify the inductive step described in the proof of Theorem 3.2 in [2]. In the second last sentence of the proof, it says, 'it follows from the inductive proof that $\mathscr{R}_n$ is of index two.' The question of how this assertion is verified was first raised by Natan and his student Ron [1]. To avoid any confusions that may arise in the future, the author of the paper [2] would like to fill in the details to show that $\mathscr{R}_n$ is of index two. Readers of this document are assumed to have familiarity with [2]. For your reference, Theorem 3.2 is stated below.

Theorem 3.2. *$\mathscr{R}_n$ forms a subgroup of the free abelian group $\mathscr{V}_n$ of index two. So its rank is the same as the rank of $\mathscr{V}_n$ and is the number of linearly independent primitive rational invariants of order $\leqslant n$.*

To show that $\mathscr{R}_n$ is of index two, we shall establish the following two lemmas.

Lemma A. *Let $D_1, \ldots, D_r$ be all the distinct non-split chord diagrams of order $n \geqslant 2$. Let $\lambda_1, \ldots, \lambda_r$ be r arbitrary integers. Then for $n > 2$, there exists a ribbon knot K_n whose additive invariants of order $< n$*

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* Tel.: +1-718-987-9351.
E-mail addresses: kaying7@hotmail.com, kayi.ng@wallstreetsystems.com (K.Y. Ng).

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are trivial and $V(K_n) = \sum_{i=1}^{r} \lambda_i V(D_i)$ *for all additive invariants V of order n. When* $n = 2$, *there exists a ribbon knot* K_2 *with* $V(K_2) = 2\lambda_1 V(D_1)$ *for all additive invariants V of order 2.*

Lemma B. *Given a knot K whose additive invariants of order* $< n$ *are trivial, there exist n-chord diagrams* $D_1, \ldots, D_r$ *and integers* $\lambda_1, \ldots, \lambda_r$ *such that* $V(K) = \sum_{i=1}^{r} \lambda_i V(D_i)$ *for all additive invariants V of order n.*

Lemma A is an extended version of Lemma 3.1 in [2] and can be proved using the same arguments as given in [2]. Lemma B is proved in [3]. We shall now make use of these two lemmas to prove Lemma C.

Lemma C. *Let X be the right-handed trefoil. Let K be an arbitrary knot and let* $n \geqslant 2$ *be an integer. Define s to be 1 if the Arf invariant of the knot K is non-zero and 0 otherwise. Then one can construct aribbon knot* K_n *so that* $V(K\#sX\#K_n) = 0$ *for all additive invariants V of order* $\leqslant n$.

Proof of Lemma C. For $n = 2$, the knot can always be represented by an integral multiple λ of the non-split 2-chord diagram D so that $V(K) = \lambda V(D)$ for all additive invariants of order 2. Write λ as $2p - s$ where p is an integer. By Lemma A, we can find a ribbon knot K_2 with $V(K_2) = -2pV(D)$ for all additive invariants of order 2. The right-handed trefoil knot has $V(X) = V(D)$ and thus, the case $n = 2$ is proved.

Suppose a ribbon knot K_n is constructed so that $V(K\#sX\#K_n) = 0$ for all additive invariants of order $\leqslant n$. Lemma B tells us that there exist integers $\lambda_1, \ldots, \lambda_r$ and$(n+1)$-chord diagrams $D_1, \ldots, D_r$ so that $V(K\#sX\#K_n) = \sum_{i=1}^{r} \lambda_i V(D_i)$ for all additive invariants of order $n + 1$. Apply Lemma A to construct a ribbon knot R whose additive invariants of order $\leqslant n$ are trivial and $V(R) = -\sum_{i=1}^{r} \lambda_i V(D_i)$ for all additive invariants of order $n + 1$. Then $V(K\#sX\#K_n\#R) = 0$ for all additive invariants of order $\leqslant n + 1$.

Lemma C implies that $V(K\#sX\#K_n) = 0$ for all rational-valued invariants of order$\leqslant n$. According to the proof of Theorem 3.2 in [2], $\mathscr{R}_n$ is a subgroup of $\mathscr{V}_n$. Thus, as an element in $\mathscr{V}_n$, we have $[K]_n \in -[sX]_n + \mathscr{R}_n$. This completes the proof that $\mathscr{R}_n$ is of index two. □

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